Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, ...

$$fib(n) = fib(n-1) + fib(n-2)$$

main problem

$$\boxed{\begin{aligned} fib(1) &= 1 \\ fib(0) &= 1 \end{aligned}}$$

$fib(5)$

$fib(4)$ $\quad$ $\boxed{fib(3)}$

$\boxed{fib(3)}$ $\quad$ $fib(2)$

Subproblem overlapping

Dynamic Programming

```
F[0] = 1
F[1] = 1
for (i = 2; i <= n; i++)
    F[i] = F[i-1] + F[i-2]
```

COIN CHANGE

C = {3, 5, 7}

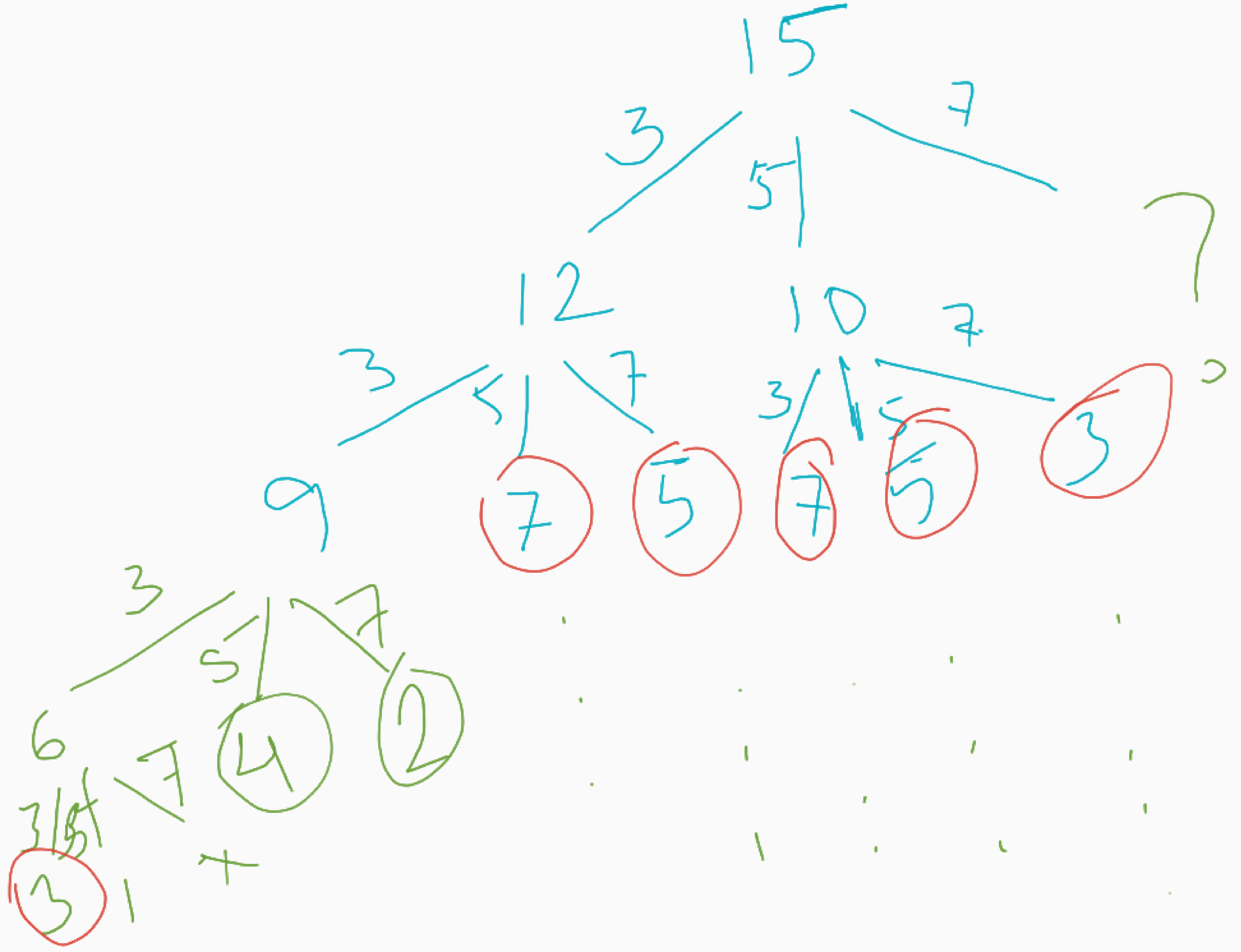
Total amount = 15

You need to make 15 with the given set of coins. YOU MAY ASSUME THAT THE COIN ARE INFINITE

But you need to make 15 with the MINIMUM number of coins

15 = 3, 3, 3, 3, 3

15 = 5, 5, 5



Using simple recursion, we would have to do A LOT of computation to find the minimum combination

There is subproblem overlapping in this scenario as well.

Optimal Solution - Dynamic Programming Approach

C = {3, 5, 7}

A = 15

Minimum No of Coins Required = 3

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
M	0	∞	∞	1	∞	1	2	1	2	3	2	3	2	3	2	3
C	0	-1	-1	3	-1	5	3	7	3	3	3	5	7	7	7	7

M[4] = M[4-3]+1 = M[1]+1 = Infinity + 1 = Infinity

Here, the index numbers represent **the remaining amount** (to be made)

R - Ci = Next Remaining Amount

M[6] = M[6-3] + 1 = M[3] + 1 = 1+1 = 2. Alternatively, M[6] = M[6-5]+1 = M[1] + 1 = Infinity. So, we pick 3.

Mathematical Representation of Coin Change ->

M means the minimum number of coins required. Ci means the coin chosen

M[5] = M[5-3] + 1 = M[2] + 1 = Infinity. SO THIS IS NOT THE MINIMUM. M[5] = M[5-5]+1 = M[0] + 1 = 1

M[0] = 0

$$M[i] = \begin{cases} \text{Infinity, when I cannot be made} \\ \text{Min}(1<=i<=|c|)\ (M[R-Ci]+1);\ \text{when I can be made and } i>=Ci \end{cases}$$

Coins chosen -> 7, 3, 5

M[7] =
M[7-3]+1=
M[4]+1=
infinity

M[7] =
M[7-5] + 1
= M[2] +1 =
infinity

M[7] =
M[7-7] + 1
= M[0] + 1
= 1

M[11] =
M[11-3] + 1
= M[8] + 1
= 3

M[11] =
M[11-5] + 1
= M[6] + 1
= 3

M[11] =
M[11-7] + 1
= M[4] + 1
= Infinity

M[8] =
M[8-3]+1 =
M[5]+1 = 2

M[8] =
M[8-5]+1 =
M[3]+1 = 2

M[8] =
M[8-7]+1 =
M[1]+1 =
Infinity

count any!

M[9] =
M[9-3] +1 =
M[6] + 1 =
3

M[9] =
M[9-5] +1 =
M[4] + 1 =
Infinity

M[9] =
M[9-7] +1 =
M[2] + 1 =
Infinity

M[10] =
M[10-3] + 1
= M[7]+1 =
2

M[10] =
M[10-5] + 1
= M[5]+1=
2

M[10] =
M[10-7] + 1
= M[3] + 1
= 2