

Formula:

(i) CPU execution time for a program = CPU clock cycles for a program \times clock cycle time.

$$(ii) \text{ clock cycle time} = \frac{1}{\text{clock rate}}$$

$$(iii) \text{ CPU execution time for a program} = \frac{\text{CPU clock cycles for a program}}{\text{clock rate}}$$

$$(iv) \text{ CPU clock cycles} = \text{Instruction count} \times \text{CPI}$$

$$(v) \text{ CPU time} = \frac{\text{Instruction count} \times \text{CPI} \times \text{clock cycle time}}{\text{clock rate}}$$

$$+ \text{(vi) cpu time} = \frac{\text{Instruction count} \times \text{CPI}}{\text{clock rate}}$$

$$\text{(iii) MIPS} = \frac{\text{Instruction count}}{\text{Execution time} \times 10^6}$$

$$= \frac{\text{Instruction count}}{\frac{\text{Instruction count} \times \text{CPI}}{\text{clock rate}} \times 10^6}$$

$$N = \frac{\text{clock rate}}{\text{CPI} \times 10^6}$$

$$* \text{ performance (p)} = \frac{\text{clock rate}}{\text{CPI}} \text{ instruction/sec}$$

$$\text{clock cycle time} = \frac{1}{\text{clock rate}}$$

$$+ \text{(ii) cpu execution time for a program} = \frac{\text{clock cycles for a program}}{\text{clock rate}}$$

$$\text{(i) cpu clock cycles} = \text{Instruction count} \times \text{CPI}$$

$$\text{(ii) cpu time} = \text{Instruction count} \times \text{CPI} \times \text{clock cycle time}$$

P → 34

Example:

A program runs in 10s on computer A, which has a 2 GHz clock. We are trying to help a computer designer build a computer B, which will run this program in 6s. The designer increase in the clock rate is possible, but this increase will affect the rest of the CPU design, causing computer B to require 1.2 times as many clock cycles as computer A for this program. What clock rate should we tell the designer to target?

Solve:

we know,

$$\text{CPU time}_A = \frac{\text{CPU clock cycles}_A}{\text{clock rate}_A}$$

$$\begin{aligned} \Rightarrow \text{CPU clock cycles}_A &= \text{CPU time}_A \times \text{clock rate}_A \\ &= (10 \times 2 \times 10^9) \text{ cycles} \\ &= 20 \times 10^9 \text{ cycles.} \end{aligned}$$

Again,

$$\text{CPU time}_B = \frac{1.2 \times \text{CPU clock cycles}_A}{\text{clock rate}_B}$$

$$\Rightarrow \text{clock rate}_B = \frac{1.2 \times \text{CPU clock cycles}_A}{\text{CPU time}_B}$$

Example:

$$= \frac{(1.2 \times 20 \times 10^9) \text{ cycles}}{6 \text{ s}}$$

$$= 4 \times 10^9 \text{ cycles/s}$$

$$= 4 \text{ GHz}$$

∴ To run the program in 6 seconds, B must have twice the clock rate of A.

(Ans:)

Solve:

we know,
CPU clock cycles

$$\text{CPU time}_A = \frac{\text{CPU clock cycles}_A}{\text{clock rate}_A}$$

⇒ CPU clock cycles_A = CPU time_A × clock rate_A

$$= (30 \times 5 \times 10^9) \text{ cycles}$$

$$= 150 \times 10^9 \text{ cycles}$$

Again,

$$\text{CPU time}_B = \frac{\text{CPU clock cycles}_B}{\text{clock rate}_B}$$

$$\Rightarrow \text{clock rate}_B = \frac{150 \times 10^9 \text{ cycles}_B}{\text{CPU time}_B}$$

P-35 Example:

Suppose we have two implementations of the same instruction set architecture. Computer A has a clock cycle time of 250 ps and a CPI of 2.0 for some program, and computer B has a clock cycle time of 500 ps and a CPI of 1.2 for the same program. Which computer is faster for this program and by how much?

Solve:

We know that, each computer executes the same number of instructions for the program; let's call this number I .

Now, CPU time for computer A:

$$\begin{aligned} \text{CPU time}_A &= I \times 2 \times 250 \text{ ps} \\ &= 500 \times I \text{ ps} \end{aligned}$$

CPU time for computer B:

		A	
		CPU time _B = $I \times 1.2 \times 500 \text{ ps}$	
		= $600 \times I \text{ ps}$.	

Clearly, computer A is faster because execution time of computer A is less than the computer B. (Ans)

The amount faster is given by the ratio of the execution times:

$$\frac{\text{CPU performance}_A}{\text{CPU performance}_B} = \frac{\text{Execution time}_B}{\text{Execution time}_A} = \frac{600 \times 1 \text{ Ps}}{500 \times 1 \text{ Ps}} = 1.2$$

\therefore computer A is 1.2 times faster than computer B. (Ans.)

P-37 Example:

A compiler designer is trying to decide between two code sequences for a particular computer. The hardware designers have supplied the following facts:

	CPI for each instruction class		
	A	B	C
CPI	1	2	3

For a particular high-level language statement, the compiler writer is

considering two code sequences that requires the following instruction counts:

code sequence	Instruction counts for each instruction		
	A	B	C
1	2	1	2
2	4	1	1

- i) which code sequence executes the most instructions?
- ii) which will be faster?
- iii) what is the CPI for each sequence?

solve:

i) sequence 1 executes $2+1+2=5$ instructions.

 " 2 " $4+1+1=6$ "

therefore, sequence 2 executes most instructions. (Ans:)

$$\text{CPI 1} = \frac{10}{2} = 5$$

$$\text{CPI 2} = \frac{6}{2} = 3$$

ii) First, find the total number of clock cycles for each sequence:

$$\text{CPU clock cycles} = \sum_{i=1}^n (\text{CPI}_i \times C_i) \quad \text{--- 1.c}$$

$$\begin{aligned} \therefore \text{CPU clock cycles}_1 &= (1 \times 2) + (2 \times 1) + (3 \times 2) \\ &= 2 + 2 + 6 \\ &= 10 \text{ cycles} \end{aligned}$$

$$\begin{aligned} \text{CPU clock cycles}_2 &= (1 \times 4) + (2 \times 1) + (3 \times 1) \\ &= 4 + 2 + 3 \\ &= 9 \text{ cycles} \end{aligned}$$

so, code sequence 2 is fastest, even though it executes one extra instruction.

(Ans)

iii) we know,

$$\text{CPI} = \frac{\text{CPU clock cycles}}{\text{Instruction count}}$$

$$\therefore \text{CPI}_1 = \frac{10}{5} = 2 \quad (\text{Ans})$$

$$\text{CPI}_2 = \frac{9}{6} = 1.5 \quad (\text{Ans})$$

1.5

P-755

Consider three different processors P₁, P₂ and P₃ which are executing on the same instruction set with the following clock rates and CPI's:

Processor	clock rate	CPI
P ₁	3 GHz	1.5
P ₂	2.5 GHz	1.0
P ₃	4 GHz	2.2

- i) Which processor has the highest performance expressed in instructions per second?
- ii) If the processors each execute a program in 10s. find the number of cycles and the number of instructions.
- iii) We are trying to reduce the execution time by 30% but this leads to an increase of 20% in the CPI. what clock rate should we have to get this time reduction?

Solve:

i) we know,

$$\text{performance (P)} = \frac{\text{clock rate}}{\text{CPI}} \text{ instructions/sec.}$$

For processor P₁:

$$\begin{aligned} \text{performance (P}_1) &= \frac{3 \times 10^9}{1.5} \text{ ins/sec} \\ &= 2 \times 10^9 \text{ ins/sec.} \end{aligned}$$

For processor P2:

$$\begin{aligned} \text{Performance}(P_2) &= \frac{2.5 \times 10^9}{1.0} \text{ ins/sec} \\ &= 2.5 \times 10^9 \text{ ins/sec} \end{aligned}$$

For processor P3:

$$\begin{aligned} \text{Performance}(P_3) &= \frac{4 \times 10^9}{2.2} \text{ ins/sec} \\ &= 1.81 \times 10^9 \text{ ins/sec} \end{aligned}$$

As the performance is inversely proportional to the time, the processor with less time performs better.

Thus, among the 3 processors, the least time is taken by the processor P2 resulting in highest performance. (Ans)

ii) Consider the CPU time for executing each program is 10s.

We know,

$$\text{Number of cycles}(P) = \text{Time} \times \text{clock rate}$$

$$\text{Number of instruction}(P) = \frac{\text{number of cycles}}{\text{CPI}} \text{ ins.}$$

Fast Processor P1:

$$\text{number of cycles (P1)} = 10 \times 3 \times 10^9 = 30 \times 10^9 \text{ (Ans.)}$$

$$\begin{aligned} \text{number of instructions (P1)} &= \frac{30 \times 10^9}{1.5} \text{ ins.} \\ &= 20 \times 10^9 \text{ ins. (Ans.)} \end{aligned}$$

Fast Processor P2:

$$\text{number of cycles (P2)} = 10 \times 2.5 \times 10^9 = 25 \times 10^9 \text{ (Ans.)}$$

$$\begin{aligned} \text{number of instructions (P2)} &= \frac{25 \times 10^9}{1} \text{ ins.} \\ &= 25 \times 10^9 \text{ ins. (Ans.)} \end{aligned}$$

Fast Processor P3:

$$\text{number of cycles (P3)} = 10 \times 4 \times 10^9 = 40 \times 10^9 \text{ (Ans.)}$$

$$\begin{aligned} \text{number of instructions (P3)} &= \frac{40 \times 10^9}{2.2} \text{ ins.} \\ &= 18.18 \times 10^9 \text{ ins. (Ans.)} \end{aligned}$$

iii) The time is decreased by 30%, so we get the new time is $\left(10 - \frac{30 \times 10}{100}\right) = 7s$

CPI is increased by 20%.

$$\begin{aligned} \text{CPI} &= \frac{120 \times \text{CPI}}{100} \\ &= 1.2 \times \text{CPI} \end{aligned}$$

$$= \text{CPI} + 0.2 \times \text{CPI}$$

we know,

$$\text{clock rate} = \frac{\text{number of instruction} \times \text{CPI}}{\text{Time}}$$

$$\text{number of instructions (p)} = \frac{\text{number of cycles}}{\text{CPI}} \text{ ins.}$$

$$\text{number of cycles (p)} = \text{Time} \times \text{clock rate.}$$

For processor P₁:

$$\text{CPI} = 1.2 \times 1.5 = 1.8$$

$$\text{number of cycles (P}_1) = 10 \times 3 \times 10^9 = 30 \times 10^9$$

$$\text{number of instructions (P}_1) = \frac{30 \times 10^9}{1.5} = 20 \times 10^9$$

$$\therefore \text{clock rate (P}_1) = \frac{20 \times 10^9 \times 1.8}{7}$$

$$= \frac{36 \times 10^9}{7}$$

$$= 5.14 \text{ GHz} \quad (\text{Ans!})$$

For processor P₂:

$$\text{CPI} = 1.2 \times 1.0 = 1.2$$

$$\text{number of cycles (P}_2) = 10 \times 2.5 \times 10^9 = 25 \times 10^9$$

$$\text{number of instructions (P}_2) = \frac{25 \times 10^9}{1.0} = 25 \times 10^9$$

$$\therefore \text{clock rate (P}_2) = \frac{25 \times 10^9 \times 1.2}{7}$$

$$= \frac{30 \times 10^9}{7} = 4.28 \text{ GHz} \quad (\text{Ans!})$$

Fast processor P₃:

$$CPI = 1.2 \times 2.2 = 2.64$$

$$\text{number of cycles (P}_3\text{)} = 10 \times 4 \times 10^9 = 40 \times 10^9$$

$$\text{number of instructions (P}_3\text{)} = \frac{40 \times 10^9}{2.2} = 18.18 \times 10^9$$

$$\therefore \text{clock rate (P}_3\text{)} = \frac{18.18 \times 10^9 \times 2.64}{7}$$

$$= \frac{47.99 \times 10^9}{7}$$

$$= 6.85 \text{ GHz. (Ans.)}$$