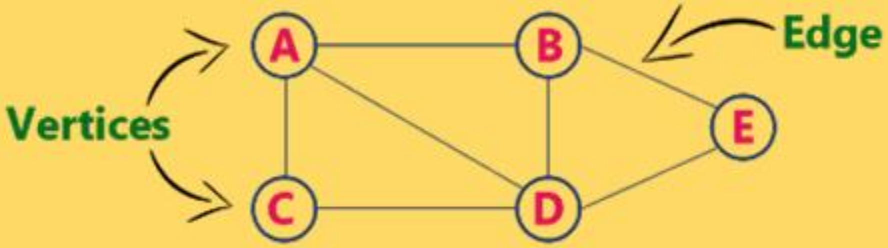
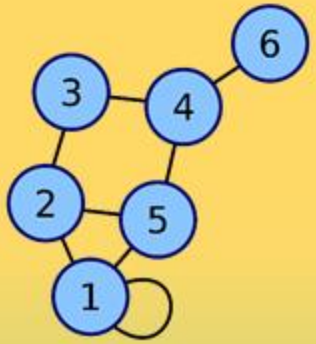
Introduction





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Graph

Outlines

- Definition of Graph
- Graph Representation
- Graph Terminology
- Different Types of Graph
- Handshaking Lemma
- Corollary using Handshaking Lemma

Definition: Graph

- A graph consists of a set of vertices (nodes) and set of edges that are used to connect these vertices.
- A graph G is defined as an ordered set G(V, E).
    - ✓ Vertex: An element of a vertex set.
    - ✓ Edge: An element of a edge set.

A Graph G(V, E) with five vertices and seven edges is shown in the following figure.

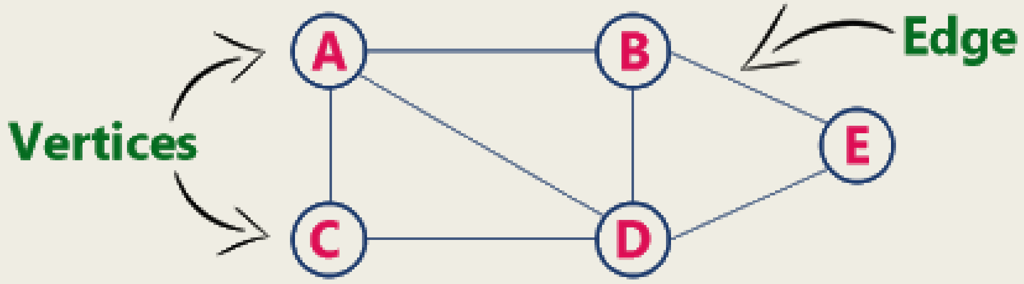


Fig. 1: Graph

Graph Representation

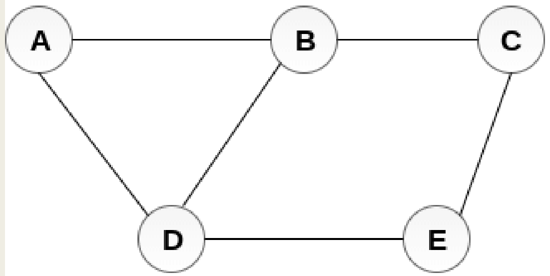
There are two ways to store Graph into the computer's memory

- Sequential Representation
- Linked Representation

Sequential Representation (adjacency matrix)

we use adjacency matrix to store the mapping represented by vertices and edges. In adjacency matrix, the rows and columns are represented by the graph vertices. A graph having n vertices, will have a dimension n x n.

An entry M_{ij} in the adjacency matrix representation of an undirected graph G will be 1 if there exists an edge between V_i and V_j.



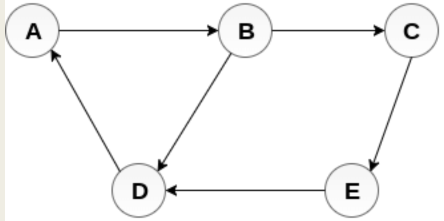
Undirected Graph

	A	B	C	D	E
A	0	1	0	1	0
B	1	0	1	1	0
C	0	1	0	0	1
D	1	1	0	0	1
E	0	0	1	1	0

Adjacency Matrix

Graph Representation: Cont…

An entry M_{ij} in the adjacency matrix representation of a directed graph G will be 1 only when there is an edge directed from V_i to V_j.

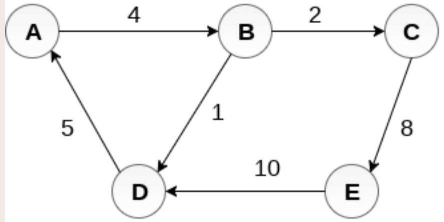


Directed Graph

	A	B	C	D	E
A	0	1	0	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	1	0	0	0	0
E	0	0	0	1	0

Adjacency Matrix

Representation of weighted directed graph is different. Instead of filling the entry by 1, the Non- zero entries of the adjacency matrix are represented by the weight of respective edges.



Weighted Directed Graph

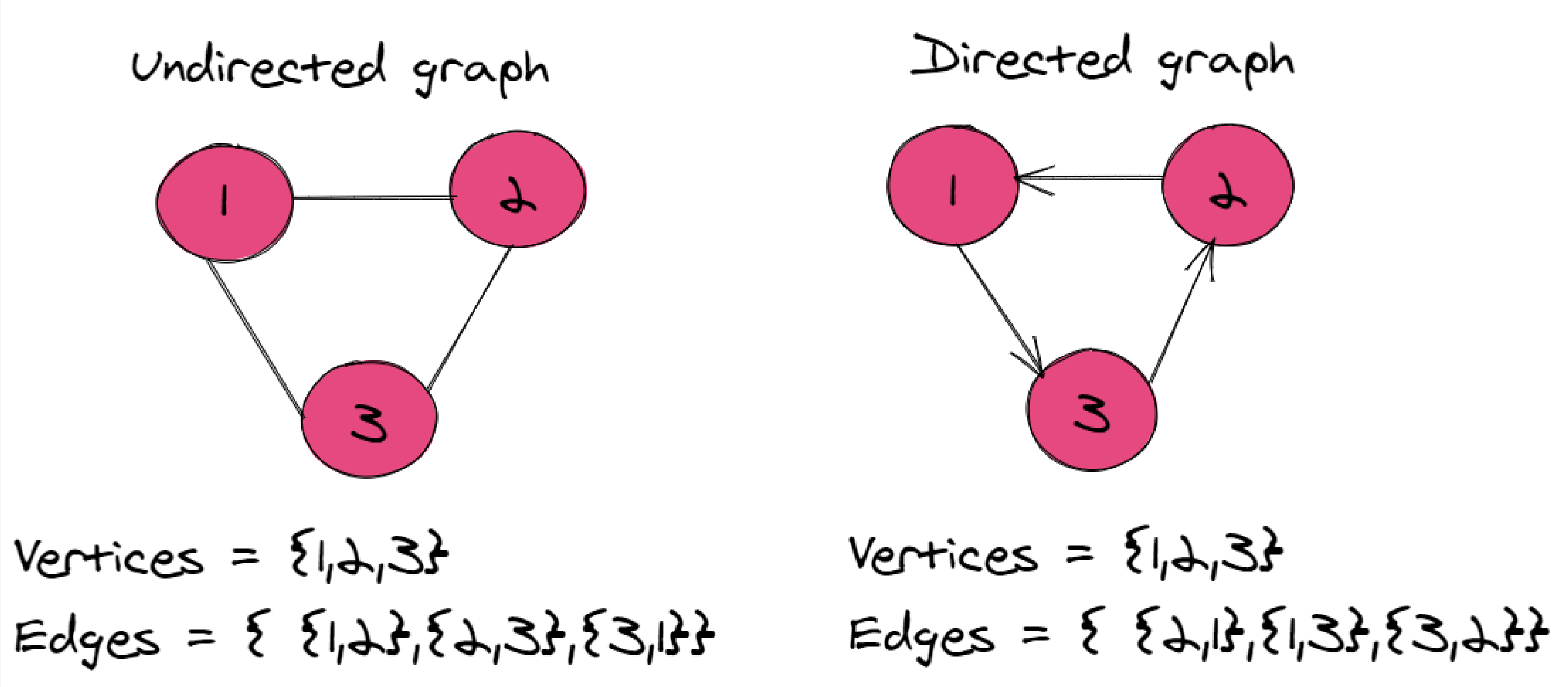
	A	B	C	D	E
A	0	4	0	0	0
B	0	0	2	1	0
C	0	0	0	0	8
D	5	0	0	0	0
E	0	0	0	10	0

Adjacency Matrix

Graph Representation: Cont…

Sequential Representation (edge list)

An edge list is a data structure used to represent a graph as a list of its edges. An (unweighted) edge is defined by its start and end vertex, so each edge may be represented by two numbers

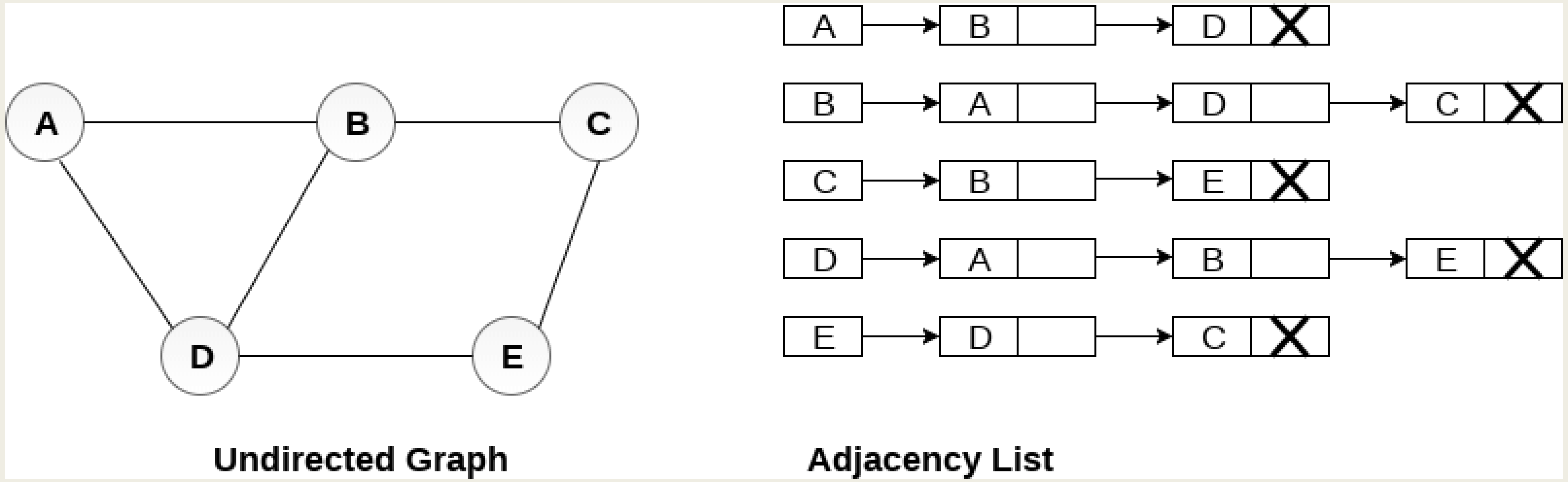


Graph Representation: Cont…

Linked Representation

An adjacency list is maintained for each node present in the graph which stores the node value and a pointer to the next adjacent node to the respective node. If all the adjacent nodes are traversed then store the NULL in the pointer field of last node of the list.

The sum of the lengths of adjacency lists is equal to the twice of the number of edges present in an undirected graph.



Graph Terminology

- ✓ Multiple or parallel edges
- ✓ Loop
- ✓ Adjacent Nodes
- ✓ Degree of a Node
- ✓ Path
- ✓ Cycle
- ✓ Tree

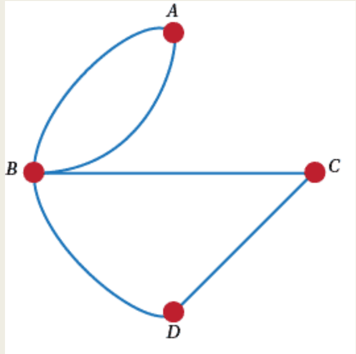


Fig. 2: Multiple edge

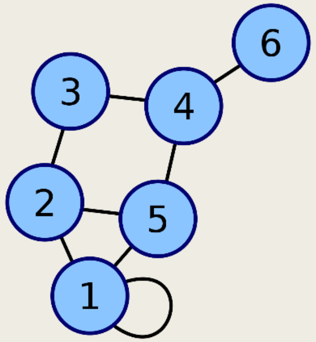


Fig. 3: Loop

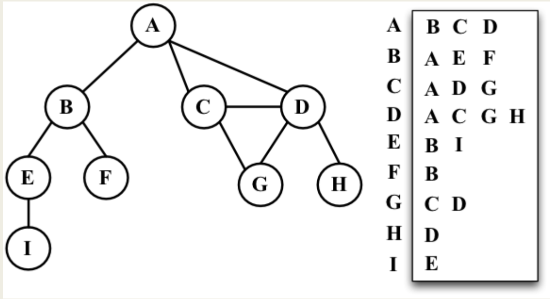


Fig. 4: Adjacent nodes

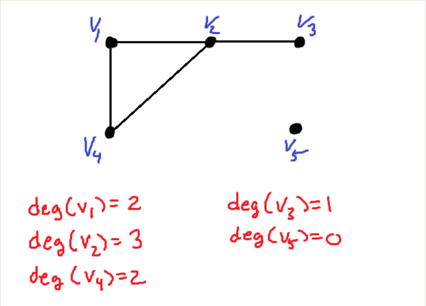


Fig. 5: Degree of a node

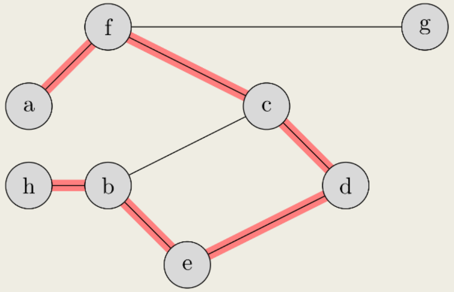


Fig. 6: Path

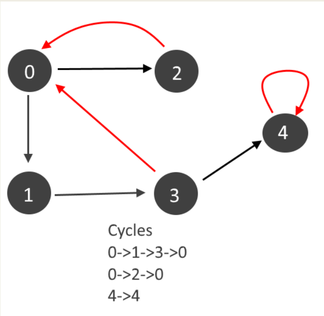


Fig. 7: Cycle

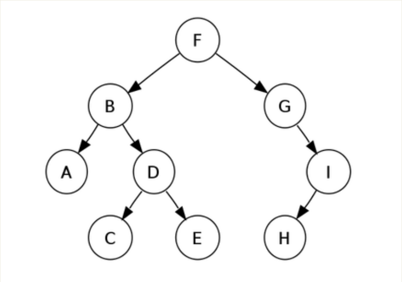


Fig. 8: Tree

Graph Terminology: Cont…

- Multiple or parallel edges: When two or more edges connecting the same two vertices, such edges are called multiple or parallel edges.
- Loop: The loops is an edges which connect a vertex to itself.
- Adjacent Nodes: If two nodes u and v are connected via an edge e, then the nodes u and v are called as neighbors or adjacent nodes.
- Degree of a Node: Number of edges that are connected/incidents with that node. A node with degree 0 is called as isolated node.
- Path: A path is a sequence of edges which joins a sequence of vertices. (no repeated edges and vertices).
- Cycle: A cycle can be defined as the path which has no repeated edges or vertices except the first and last vertices.
- Tree: A connected graph with no cycle.

Types of Graph

- **Undirected graph**: All edges must be undirected.
- **Directed graph**: All edges must be directed.
- **Simple graph:** No loops or multiple edges.
- **Multigraph**: Contains multiple edges or loops or both.
- **Mixed graph:** Contains both directed and undirected edges.

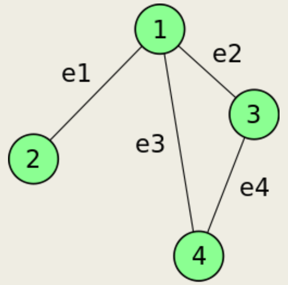


Fig. 9: Undirected graph

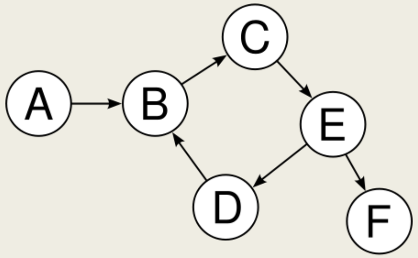


Fig. 10: Directed graph

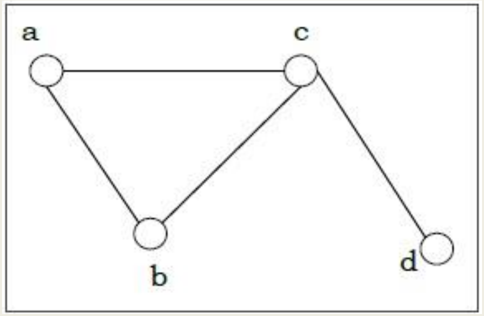


Fig. 11: Simple graph

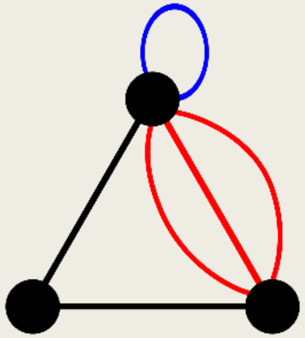


Fig. 12: Multi graph

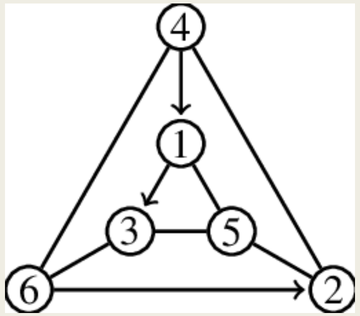


Fig. 13: Mixed graph

Types of Graph: Cont…

- Connected graph: Some path exists between every two vertices.

- Weighted graph: Each edge is assigned with some data such as cost, length.

- Complete graph: Every node is connected with all other nodes.

  - It is denoted by K_n, (n = Number of vertices or nodes)
  - Each vertex has (n-1) degree.
  - It contain n(n-1)/2 edges.

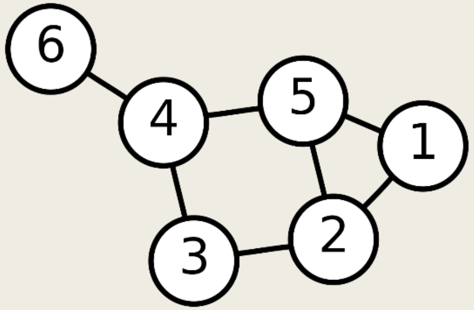


Fig. 14: Connected graph

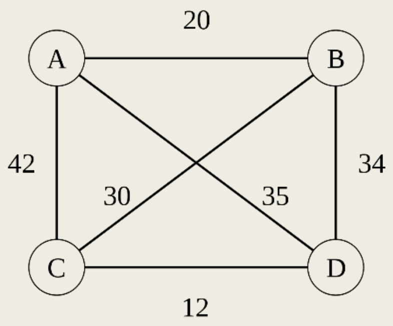


Fig. 15: Weighted graph

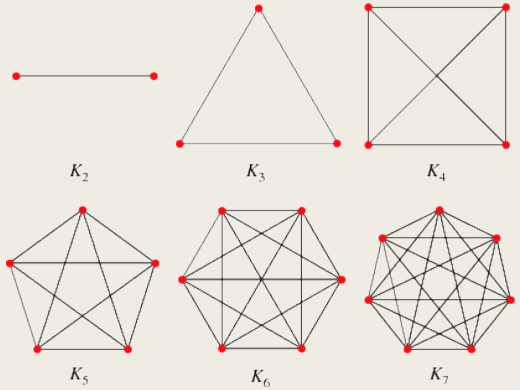
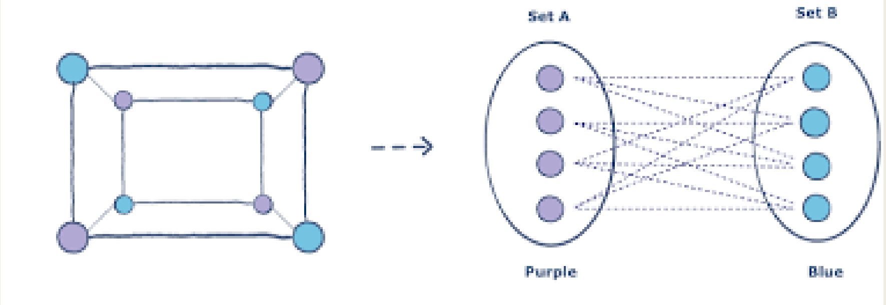


Fig. 16: Complete graph

Types of Graph: Cont…

- A simple graph G(V,E) is bipartite if V can be partitioned into two disjoint subsets V_1 and V_2 such that every edge connects a vertex in V_1 and a vertex in V_2.



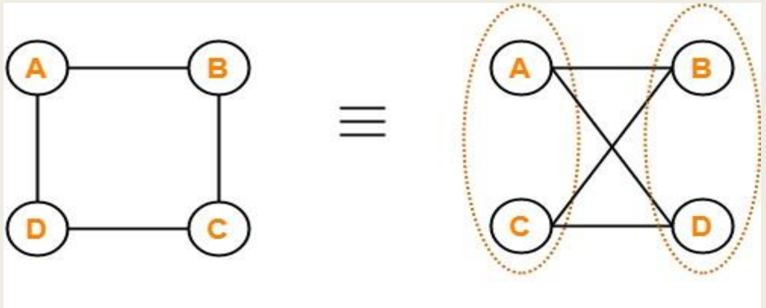


Fig. 17: Bipartite graph

- A complete bipartite graph is a bipartite graph in which each vertex in the first set is joined to every single vertex in the second set.
    - It is denoted by $K_{m,n}$ where, m is the first vertices set and n is the second vertices set.
    - $K_{m,n}$ has mn edges.

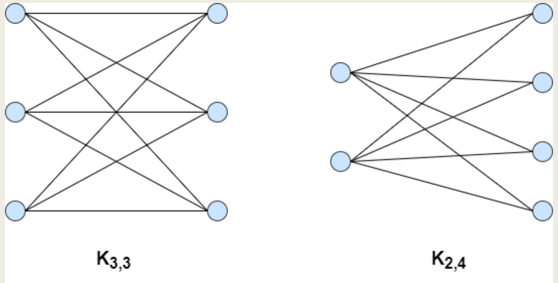


Fig. 18: Complete bipartite graph

Handshaking Lemma

In any graph, the sum of all the vertex degree is equal to twice the number of edges.

or

If G(V,E) is an undirected graph then,

$$\sum_{v \in V} \deg(v) = 2|E|$$

Proof:

Since each edge has two ends, it must contribute exactly two (2) to the sum of the degrees.

Handshaking Lemma: Example

Suppose, an undirected graph is:

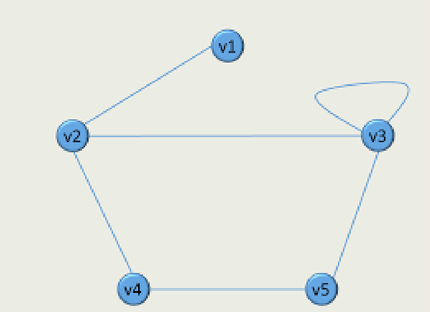


Fig. 19: Undirected graph

We know, deg(v)= Number of incident edges + 2*loop edges
Here, number of edges = 6

$$\sum \deg(v) = \deg(v1) + \deg(v2) + \deg(v3) + \deg(v4) + \deg(v5)$$

$$\sum \deg(v) = 1 + 3 + 4 + 2 + 2$$

$$\sum \deg(v) = 12 = 2 * (\text{number of edges})$$

Corollary

In an undirected graph, there must be an even number of vertices that have odd degree.

Proof:

Let, V_1 be the vertices of odd degree and V_2 be the vertices of even degree in an undirected graph G=(V,E) with e edges.

We know, By handshaking theorem,

$$\sum_{v \in V} \deg(v) = 2|e|$$

$$\sum_{v \in v_1} \deg(v) + \sum_{v \in v_2} \deg(v) = 2|e|$$

Corollary: Cont…

$$\sum_{v \in v_1} \deg(v) + Even\ number = Even\ number$$

$$\sum_{v \in v_1} \deg(v) = Even\ number - Even\ number$$

$$\sum_{v \in v_1} \deg(v) = Even\ number$$

Hence, the number of vertices of odd degree must be even.

Corollary: Example

An undirected graph is given below:

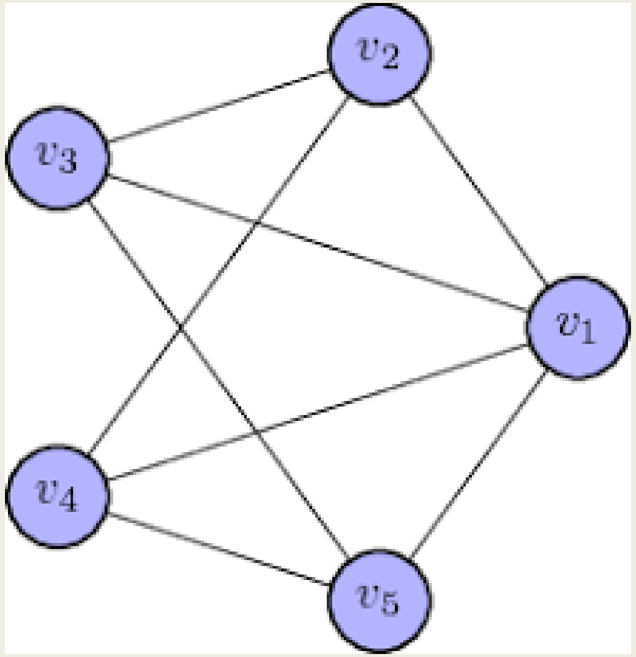


Fig. 20: Undirected graph

Here,

Vertices, V	v_1	v_2	v_3	v_4	v_5
deg (v)	4	3	3	3	3

So, the number of vertices of odd degree is 4 that is even.

Thank You

Any Question ?