

0/1 KnapSack Problem Statement

Item	Weight	Benefit
1	2	15
2	3	12
3	2	20
4	6	25

Max Sack Size, $W = 5$ kg

You can take the amount of items lesser than or equal to the sack size. You cannot pick the same item twice. But when you pick an item, you take the whole weight.

How can we maximize our benefit by taking such objects that are not larger than W ?

Picked Objects weight, $\sum w_i \leq W$

**We will solve
this in a
dynamic
programming
approach.**

Item	Weight	Benefit
1	2	15
2	3	12
3	2	20
4	6	25

Current Sack Size

no sack

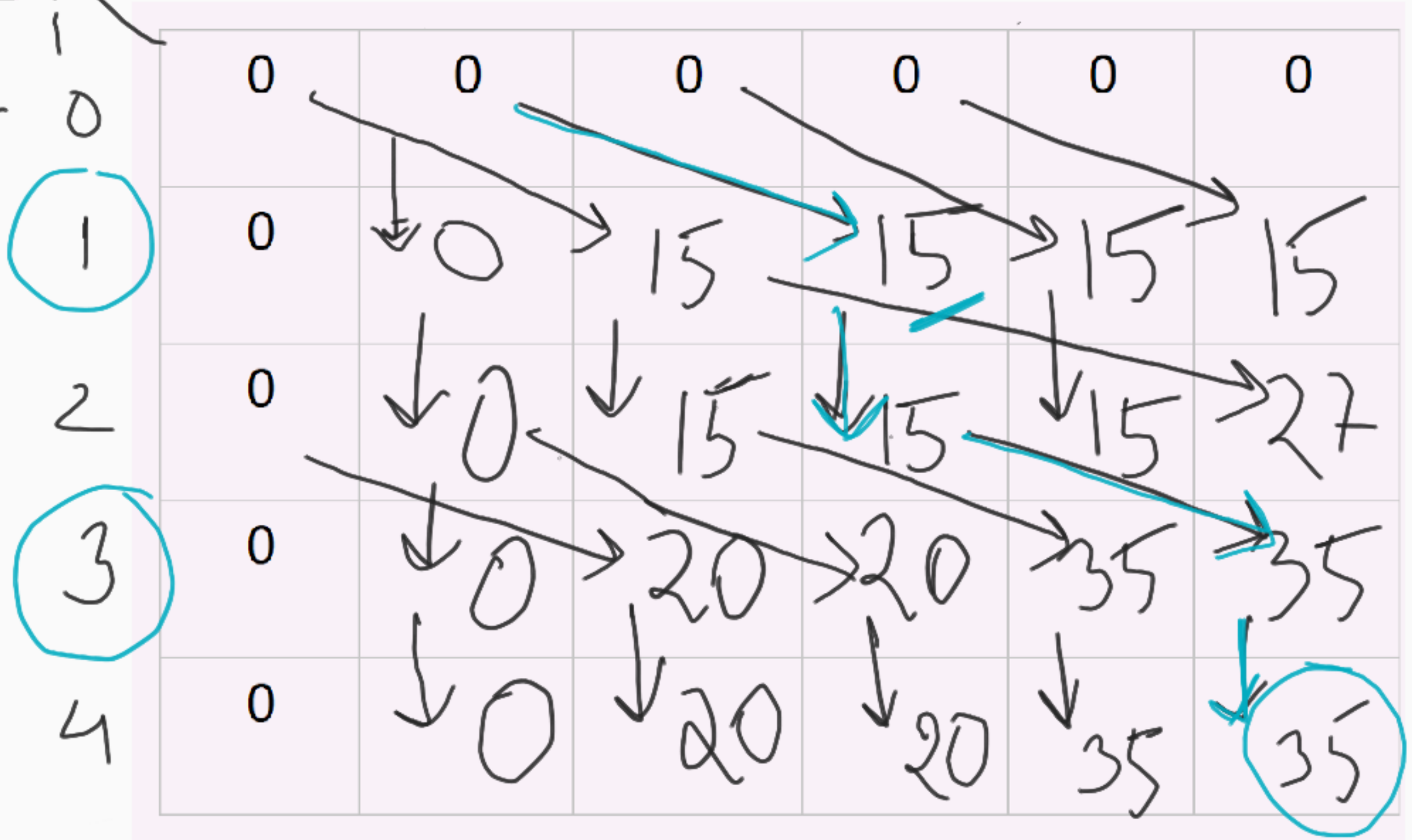
0 1 2 3 4 5

Max Sack Size = 5

no items considered

Max Benefit = 35

Items picked = 1, 3



Max Sack Size = W
 $w_i + w_p \leq W$

If we take
Item i , we can
consider this
formula ->

Current
item

Current
weight

Current
Sack
Capacity

ith item
not taken
 $\Rightarrow 0$

ith item
taken $\Rightarrow 1$

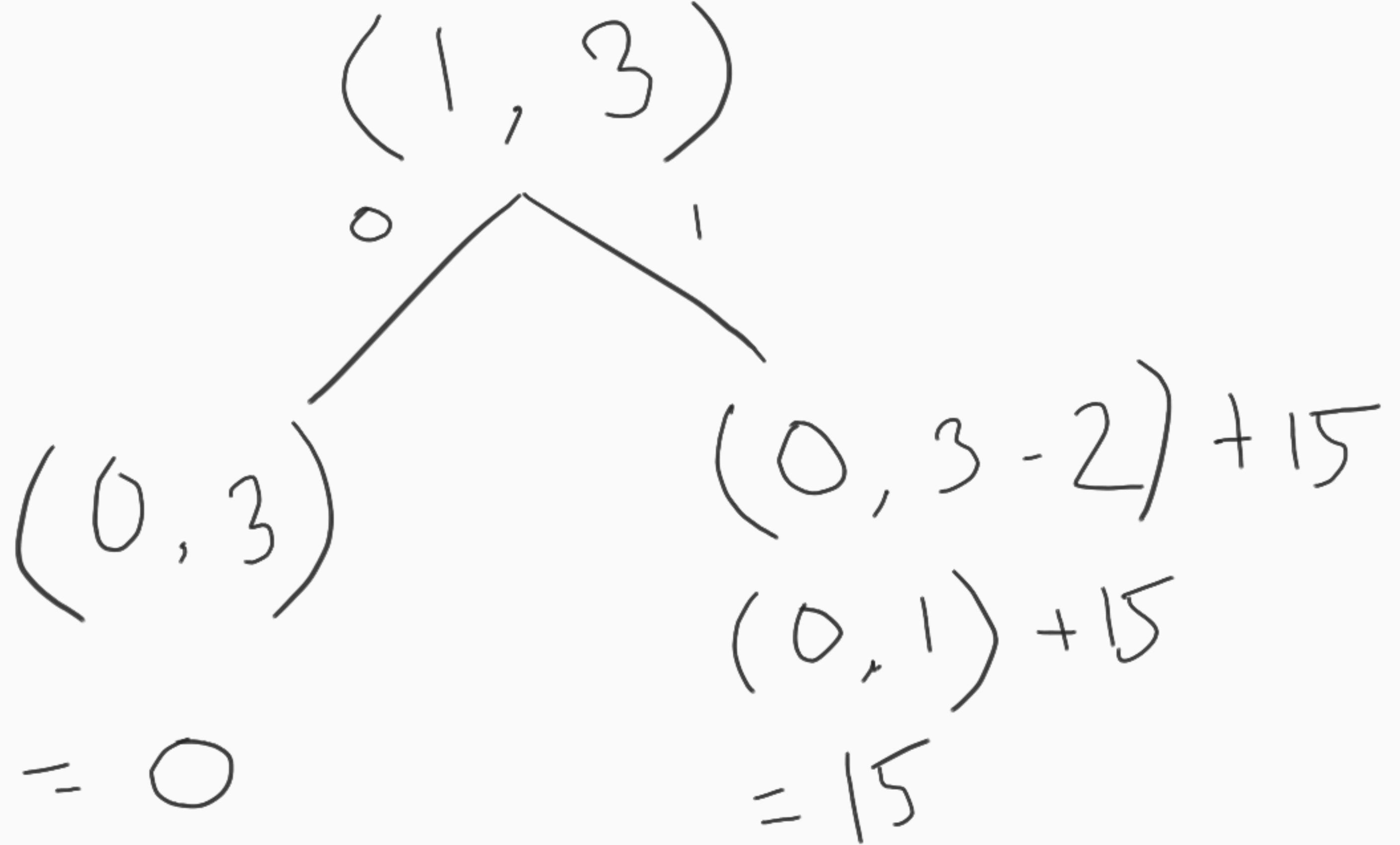
$$\left[(i-1, w) \right] \quad \left[(i-1, w - w_i) \right] + b_i$$

$$\begin{array}{c}
 \begin{array}{cc}
 i & w \\
 (1, 1) \\
 \swarrow \quad \searrow \\
 (0, 1) & (0, 1-2) + 15 \\
 & (0, -1) + 15 \\
 & = \text{invalid}
 \end{array} \\
 = 0 \\
 \swarrow \\
 \text{Max?}
 \end{array}$$

item $(1, 2)$

$$(0, 2) \\ = 0$$

$$(0, 2-2) + 15 \\ (0, 0) + 15 \\ = 15 \checkmark \text{ MAX}$$



$(2, 1)$



$(1, 1)$

0

$(1, \underline{1-3}) + 12$

= invalid

$(2, 2)$

0

1

$(1, 2)$

$(1, 2-3) + 12$

invalid

15



$(2, 3)$

0

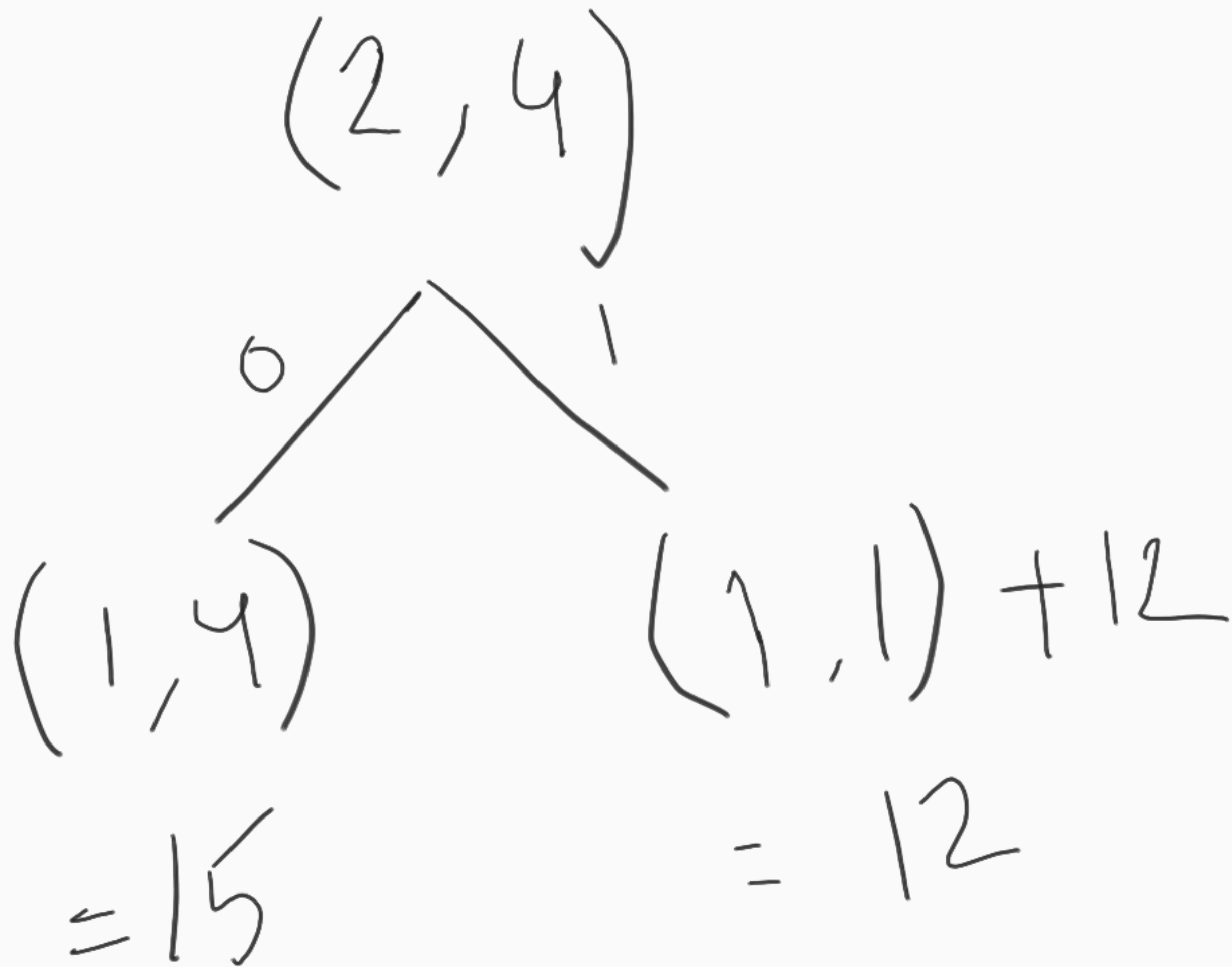
1

$(1, 3)$

$= 15$

$(1, 0) + 12$

$= 12$



$$(2, 5)$$

0

1.

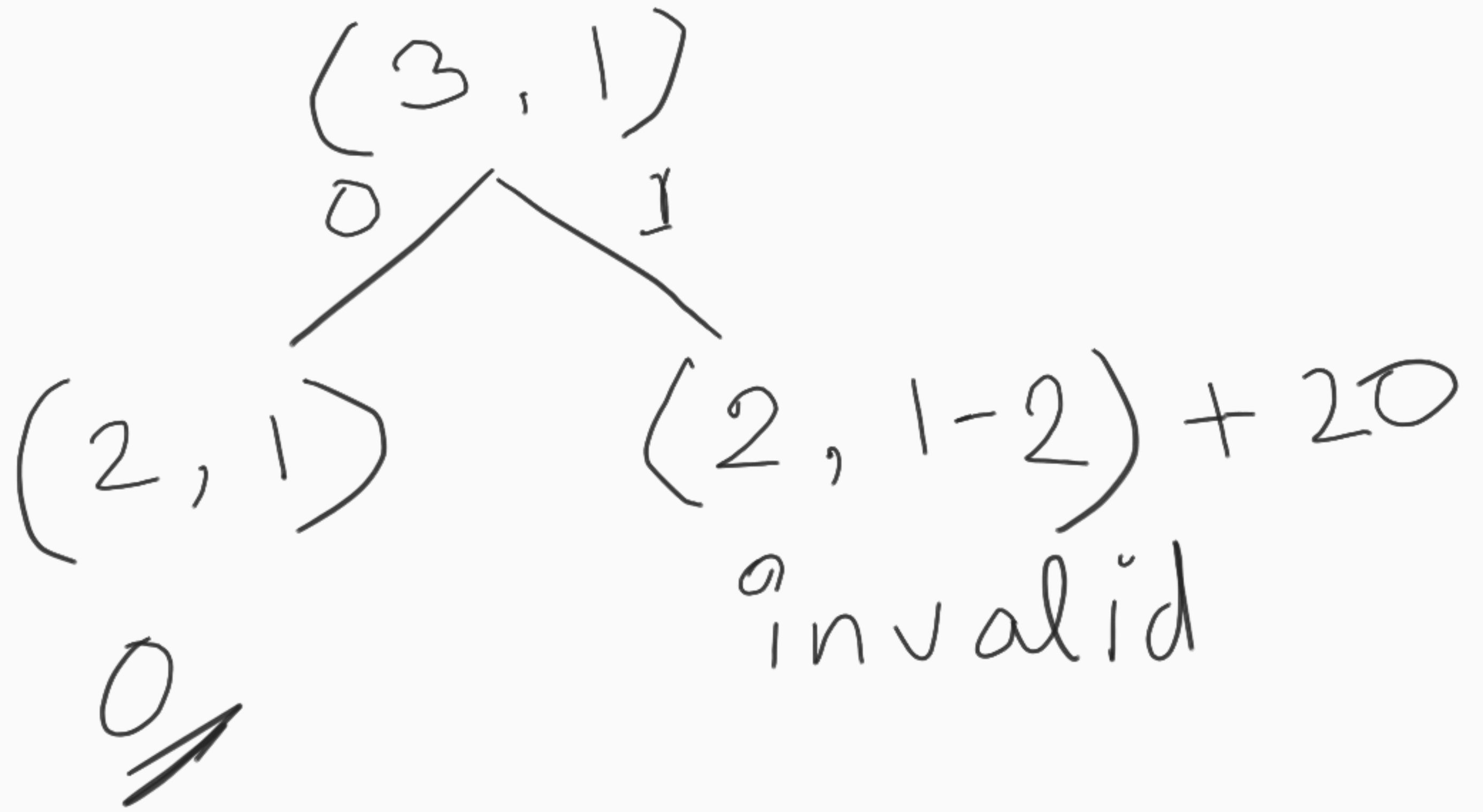
$$(1, 5)$$

$$= 15$$

$$(1, 2) + 12$$

$$= 15 + 12$$

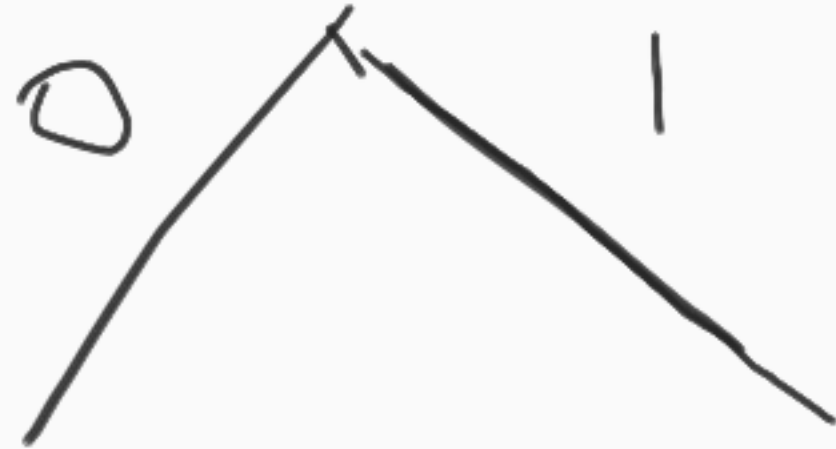
$$= 27$$



$$\begin{array}{l}
 (3, 2) \\
 \swarrow \quad \searrow \\
 \begin{array}{l}
 0 \\
 (2, 2)
 \end{array}
 \end{array}
 \begin{array}{l}
 1 \\
 (2, 2-2) + 20 \\
 = \underline{(2, 0)} + 20 \\
 = 0 + 20 + \boxed{20} \\
 \checkmark
 \end{array}$$

$$= 15$$

(3, 3)



(2, 3)

= 15

(2, 3-2) + 20

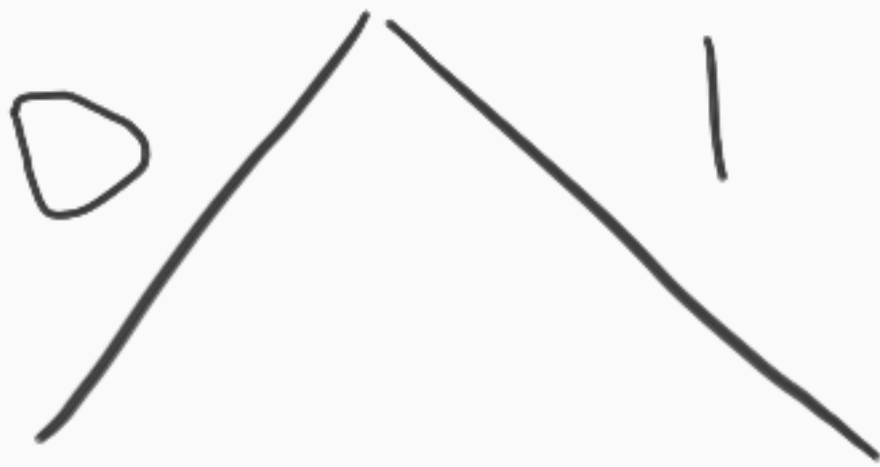
= (2, 1) + 20

= 0 + 20 = 20

$$\begin{array}{cc}
 (3, 4) & \\
 \swarrow 0 & \searrow 1 \\
 (2, 4) & (2, 2) + 20 \\
 = 15 & = 15 + 20 \\
 & = 35
 \end{array}$$

$$\begin{array}{cc}
 (3, 5) & \\
 \swarrow 0 & \searrow 1 \\
 (2, 5) & \underline{(2, 3) + 20} \\
 27 & 15 + 20 \\
 & = 35
 \end{array}$$

$(4, 1)$



$(3, 1)$

$(4, 1-6)+25$

invalid